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Charged kaon condensation in high density quark matter

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Abstract

We show that at asymptotically high densities the "color-flavor-locked + neutral kaon condensate" phase of QCD develops a *charged* kaon condensate through the Coleman–Weinberg mechanism. At densities achievable in neutron stars a charged kaon condensate forms only for some (natural) values of the low energy constants describing the low-lying excitations of the ground state. © 2002 Published by Elsevier Science B.V.

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It was realized a long time ago [1–3] that the attraction between two quarks close to the Fermi surface in high density strongly interacting matter leads to the formation of Cooper pairs of quarks and the spontaneous breaking of color symmetry. This phenomenon was more recently studied using Nambu–Jona-Lasinio models and renormalization group methods [4–7], instanton models [8] and, at asymptotically high densities where a weak coupling expansion is valid, perturbative QCD [9–11]. The main lessons learned in these studies were that (i) the gap can be large, up to 100 MeV, (ii) in the case of three-flavors of quarks with the same mass the ground state is the so-called color-flavor-locked (CFL) state, and chiral symmetry remains broken at arbitrarily high densities and (iii) the low-lying excitations carry the quantum numbers of the pseudo-scalar octet familiar from the zero density case (plus two other scalars related to the spontaneous breaking of baryon number and axial charge) and (iv) a form of electromagnetism survives: a com-

bination of the photon and the eight gluon is not "Higgsed" and remains massless in the CFL phase. The equation of state is left nearly unchanged by the pairing. A number of electromagnetic and transport properties of quark matter are, however, sensitively dependent on what phase the system finds itself and what the low energy excitation are. This dependence provides a unique opportunity to study quark matter in the interior of neutron stars (or rule out its existence).

The fact that the low-lying excitation of the ground state are very similar to the low-lying excitations of the vacuum (pions, kaons, ...) allows us to studied small perturbations around this ground state with the techniques of chiral perturbation theory. This brings about two advantages. At asymptotic high densities where perturbative QCD is valid it organizes perturbative calculations that would be very complicated otherwise. More importantly, it provides a method to systematically expand around the CFL phase in inverse powers of the density and/or the gap, which is particularly useful if the use of perturbative QCD is not legitimate. One of the important perturbations around the CFL phase is the presence of realistic quark masses.

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In the case of free quarks it is easy to determine the response of the system to these masses. As the mass of the strange quark increases towards its realistic value its density decreases in such a way that its Fermi energy equals the Fermi energy of the up quark (plus the Fermi energy of the electrons) and the weak decay $s \to u + e^- + \nu_e$ becomes forbidden. Charge neutrality is guaranteed by the presence of electrons (we assume that the neutrinos leave the system). The interacting case may be qualitatively different. If the interactions are such that quarks of different flavors are paired, the change of flavor caused by flavor changing decays would result in two unpaired quarks, what is not energetically favorable. As a result, the system is rigid against small enough flavor asymmetries. That is what happens in two-flavor QCD, where up and down quarks (of two of the three colors) are paired. An asymmetry in mass or chemical potential between the flavors causes little change in the ground state if they are small enough, the small change coming entirely from the unpaired quarks of the third color [12]. One might think that the same effect occurs in the three-flavor, CFL phase, since there all quarks are paired with quarks of different flavors. The CFL phase has, however, a another way of responding to mass asymmetries that costs little energy but is not available in the free or two-flavor system: it can condense mesons carrying strangeness, that are particularly light [13]. This was demonstrated in [13] on very general grounds and, in the case of weak coupling, through explicit computations of the response function to mass asymmetries. For realistic values of the quark masses and densities it was found that the K^0 is the meson that condenses and this ground state will be referred to from now on as the "CFL + K^0" phase. For more general asymmetries, including asymmetries on chemical potential present before weak equilibrium is achieved and/or neutrinos leave the system, a rich phase diagrams results, with kaonic, pionic, neutral and charged condensates forming with different values of the parameters [19]. The pattern of symmetry breaking caused by the K^0 condensation (in the isospin limit) is the same one found in the standard electroweak model $SU_I(2) \times U_Y(1) \to U_Q(1)$ ($I = $ isospin, $Y = $ hypercharge and $Q = $ modified electric charge). Due to the lack of Lorentz symmetry, only two, and not three, Goldstone bosons are generated, one neutral and another charged [20,21]. There are stable, supercon-

ducting topological vortices [22,23] and almost stable non-topological domains walls in the "CFL + K^0" phase [24]. In the CFL phase there is an equal number of quarks of the three flavors, and the system achieves electrical neutrality in the absence of any electrons, making it a perfect insulator [25]. The presence of the K^0 condensate, being neutral, does not change this situation. A *charged* kaon condensate, however, would change quark matter from a perfect insulator to a (electrical) superconductor. It is a generic feature of charged massless scalars that the strong long wavelength fluctuations of the gauge field lead to condensation of the scalar field (Coleman–Weinberg mechanism [26]). In the CFL + K^0 phase there is one *almost* massless charged scalar field. Its mass comes from isospin breaking contributions coming from the quark mass difference and electromagnetic mass effects. In this Letter we consider the competition between the isospin breaking mass terms and the fluctuations of the electromagnetic field in order to determine the fate of the charged kaons and of the possibility of a (electromagnetic) superconducting phase in quark matter.

In the absence of quark masses, the symmetry breaking pattern generated by diquark condensation in the CFL phase is [6] $SU_c(3) \times SU_L(3) \times SU_R(3) \times U_B(1) \times U_A(1) \to SU_{c+L+R}(3) \times Z_2$. The electromagnetic $U_Q(1)$ is a subgroup of the chiral group $SU_L(3) \times SU_R(3)$ and the there is a surviving local $U_{\widetilde{Q}}(1)$ "electromagnetism" in $SU_{c+L+R}(3)$ that is a combination of the photon and one of the gluons. The axial $U_A(1)$ is only an approximate symmetry of high density QCD due to the instanton suppression in the medium. This symmetry breaking pattern implies the existence of two singlet Goldstone bosons associated with the broken baryon number and axial symmetry, and an octet of pseudoscalars. The singlets will not play a role in our analysis and will be dropped from now on. At low (excitation) energies below the gap, QCD is equivalent to the most general theory of an octet of pseudoscalars and photons with the same symmetries of QCD. This theory has been extensively analyzed [27–31] recently. The leading terms of its Lagrangian are

$$\mathcal{L} = \frac{\epsilon}{2}\vec{E}^2 - \frac{1}{2}\vec{B}^2 + \frac{f^2}{4}\mathrm{Tr}\big[D_0 \Sigma^\dagger D_0 \Sigma - v^2 \nabla \Sigma^\dagger \nabla \Sigma\big]$$
$$+ \frac{a\Delta^2}{8\pi}\mathrm{Tr}\big[\widetilde{\mathcal{M}}\big(\Sigma + \Sigma^\dagger - 2\big)\big]$$

$$+ b\tilde{\alpha} f^2 \Delta^2 \mathrm{Tr}\big[\Sigma^\dagger, Q\big][Q, \Sigma]$$
$$+ c\tilde{\alpha}^2 f^4 \big(\mathrm{Tr}\big[\Sigma^\dagger, Q\big][Q, \Sigma]\big)^2 + \cdots, \qquad (1)$$

where μ is the baryon chemical potential, Δ is the gap,

$$D_0\Sigma = \partial_0\Sigma + \frac{i}{2\mu}\big[\mathcal{M}^2, \Sigma\big] - i\tilde{e}A_0[Q, \Sigma],$$

$$\vec{D}\Sigma = \vec{\nabla}_0\Sigma - i\tilde{e}\vec{A}[Q, \Sigma],$$

$$\tilde{e} = eg\sqrt{3}/\sqrt{3g^2 + 4e^2}$$

(g = strong coupling constant, e the electron charge), $\tilde{\alpha} = \tilde{e}^2/4\pi$, $\mathcal{M}$ and Q are the quark mass and charge matrix, $\widetilde{\mathcal{M}} = \mathcal{M}^{-1}\det\mathcal{M}$ and $\Sigma = e^{i\pi^A \frac{\lambda^A}{\sqrt{2}f}}$. A few comments are in order here. The electromagnetic field in Eq. (1) are the rotated fields that remain massless in the CFL phase. The low energy constants f, v, a, b, c, ϵ can, in principle, be determined from QCD. In practice, this can be done only in the asymptotic limit where perturbation theory is valid. At lower densities one can estimate their values by looking at their variation with the cutoff of the effective theory (see below). The dielectric constant ϵ was computed in [27] where it was found that $\epsilon = 1 + \frac{8}{9\pi}\frac{\tilde{\alpha}\mu^2}{\Delta^2}$. The magnetic permeability was argued to be unchanged from the vacuum value because the diquark condensate carries no spin. We will assume this to be true even outside the perturbative QCD regime. The values of f, v were also determined in perturbation theory [31] to be $v = 1/\sqrt{3}$ and $f = (21 - 8\ln(2))/36\pi^2$. After some controversy [29–33] the value of a seems to have settled at $a = 12/\pi$ [32, 33]. Finally, the value of the gap is estimated to be around 50–100 MeV in phenomenological models at $\mu \simeq 500$ MeV and is given in perturbation theory by $\Delta = 512(2/N_F)^5/2\pi^4/g^5 e^{-\frac{3\pi^2}{\sqrt{2}g}}$ [9,10], although large corrections from higher orders are expected [14–16]. The coefficient of the term quartic in $\mathcal{M}$ in Eq. (1) is not a free parameter because it is related by an approximate local symmetry of high density QCD to the kinetic term [13]. Terms violating this symmetry (like the other mass term in Eq. (1)) are suppressed by extra powers of $1/\mu$. The electromagnetic coefficients b and c have not yet been computed in perturbation theory but we will estimate them below. The terms implied by the dots in Eq. (1) are further suppressed

by powers of momenta or meson masses in units of the cutoff $\Lambda \simeq 2\Delta$ or extra powers of $\tilde{\alpha}$.

For values of Δ satisfying

$$\cot\left(\frac{\phi}{\sqrt{2}f}\right)\frac{f^2}{\mu^2}\frac{\pi(m_u + m_d)m_s}{a - \frac{64\pi b\tilde{\alpha}f^2}{m_s(m_d - m_u)}} < \Delta^2 < \frac{f^2}{\mu^2}\frac{\pi}{a}\frac{m_s^3}{m_u}, \qquad (2)$$

the minimum of the potential is found at [13]

$$\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\left(\frac{\sqrt{2}\phi}{f}\right) & i\sin\left(\frac{\sqrt{2}\phi}{f}\right) \\ 0 & i\sin\left(\frac{\sqrt{2}\phi}{f}\right) & \cos\left(\frac{\sqrt{2}\phi}{f}\right) \end{pmatrix}, \qquad (3)$$

with

$$\cos\left(\frac{\sqrt{2}\phi}{f}\right) = \frac{am_u\Delta^2\mu^2}{\pi f^2(m_s - m_d)^2(m_s + m_d)}, \qquad (4)$$

describing a K^0 condensate (for reasonable values of the parameters $\cot\left(\frac{\phi}{\sqrt{2}f}\right)$ is nearly one). The upper limit in Eq. (2) is the maximum value of Δ for K^0 condensation and the lower limit marks the onset of K^+ condensation. At very high μ the numerical value of the range in Eq. (4) is fairly independent of the chemical potential μ. Taking the perturbative QCD values of f, a, v and ϵ, $m_u = 4$ MeV and $m_s = 150$ MeV this range is $(2\text{ MeV})^2 < \Delta^2 < (120\text{ MeV})^2)$ and thus, most likely, the real world case will correspond to the "CFL + K^0" phase. Notice that in the CFL phase the kaons are the lightest mesons and so are the first to condense under perturbations. In the isospin limit neutral and charged kaons are degenerate. The reason for the neutral kaons to condense are their slightly smaller mass due to the $m_d - m_u$ quark mass difference and electromagnetic corrections and the fact that the presence of a charged kaon condensate implies the presence of electrons to guarantee charge neutrality, which raises the energy of this state compared to the CFL + K^0 phase.

This conclusion may be changed by the inclusion of photon loops and electromagnetic interaction terms that, by consistence, must be included together. Let us now compute the one photon loop contribution to the effective potential and leave the discussion of the conditions under which it is important for later.

It is convenient to use a modified Landau gauge fixing procedure, that is, we add a term $-1/2\xi(\partial_0 A^0 +$
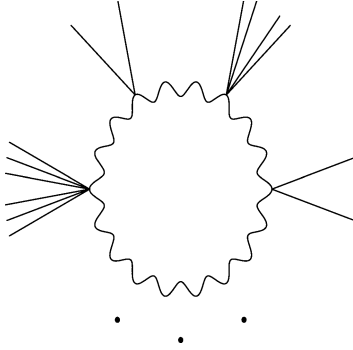
Fig. 1. Graphs giving rise to the one loop effective potential in the Landau gauge. Solid lines are mesons, wiggly line are photons.

$v^2 \nabla \vec{A})^2$ to the Lagrangian, taking the limit $\xi \to 0$. The price payed by having a complicated propagator that breaks Lorentz invariance is compensated by the fact that all zero external momentum one-loop diagrams involving a meson propagator vanish, since the photon propagator satisfies, in this gauge

$$
p_\mu V^{\mu\nu} D_{\nu\lambda}(p) = 0, \quad V^{\mu\nu} = \mathrm{diag}(1, v^2, v^2, v^2).
\tag{5}
$$

The one-loop effective potential is then given by the sum of diagrams shown in Fig. 1:

$$
\begin{aligned}
V_{\text{1-loop}} &= \sum_{n=1}^{\infty} \frac{1}{2n} \left(-\frac{\tilde{e}^2 f^2 \, \mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma]}{2} \right)^n \\
&\quad \times \lambda^{-\eta} \int \frac{d^D p}{(2\pi)^D} \, \mathrm{Tr} \underbrace{D(p) V D(p) V \cdots}_{n} \\
&= -\frac{i}{2} \lambda^{-\eta} \int \frac{d^D p}{(2\pi)^D} \\
&\quad \times \left[2 \ln \left(1 - \frac{v^2 \tilde{e}^2 f^2 \, \mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma]/2}{\epsilon p_0^2 - p^2} \right) \right. \\
&\quad \left. + \ln \left(1 - \frac{v^2 \tilde{e}^2 f^2 \, \mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma]/2}{\epsilon(p_0^2 - v^2 p^2)} \right) \right] \\
&= \frac{1}{64\pi^2} \left(\frac{2v^4}{\sqrt{\epsilon}} + \frac{v}{\epsilon^2} \right) \\
&\quad \times \left(\frac{\tilde{e}^2 f^2 \, \mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma]}{2} \right)^2 \lambda^{-\eta}
\end{aligned}
$$

$$
\times \left[\left(\frac{2}{\eta} + \gamma - \ln 4\pi - \frac{3}{2} \right) \right.
$$
$$
\left. + \ln \left(\frac{v^2 \tilde{e}^2 f^2 \, \mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma]}{2} \right) \right],
\tag{6}
$$

where we work in $D = 4 + \eta$ dimensions and λ is an arbitrary renormalization scale. The ultraviolet divergences are absorbed in the terms proportional to b and c in Eq. (1), which suggests that the natural values for these (renormalized) constants at the cutoff scale $\lambda \simeq 2\Delta$ are

$$
b(\lambda \simeq 2\Delta) = \frac{\bar{b}}{8\pi} \left(\frac{2v^2}{\sqrt{\epsilon}} + \frac{1}{v\epsilon^2} \right),
$$
$$
c(\lambda \simeq 2\Delta) = \frac{\bar{c}}{16} \left(\frac{2v^4}{\sqrt{\epsilon}} + \frac{v}{\epsilon^2} \right),
\tag{7}
$$

where $\bar{b}$ and $\bar{c}$ are numbers of order one.[1] In addition we expect on physical grounds that $\bar{b} < 0$, what guarantees a positive electromagnetic contribution to the mass square for the mesons (the photon loop contribution vanishes in the Landau gauge). This estimate of the coefficient b agrees with the ones in [17,18] (where no attempt was made to count factors of 4π) and [19] (where no attempt was made to count factors of ϵ or v). Other electromagnetic terms not renormalized at one loop order are assumed to be suppressed. Also, the contribution from meson loops is proportional to $(m_K/f)^4 \sim (\Delta/\mu)^4((m_u + m_d)m_s/f)^2$ and is strongly suppressed. Finally, the effective potential including the one photon loop correction becomes

$$
\begin{aligned}
V_{\text{eff}} &= -\frac{a\Delta^2}{8\pi} \, \mathrm{Tr} \, \widetilde{\mathcal{M}} (\Sigma + \Sigma^\dagger - 2) \\
&\quad - \frac{f^2}{16\mu^2} \, \mathrm{Tr}[\Sigma^\dagger, \mathcal{M}^2][\mathcal{M}^2, \Sigma] \\
&\quad - \bar{b} \left(\frac{2v^2}{\sqrt{\epsilon}} + \frac{1}{v\epsilon^2} \right) \frac{\tilde{\alpha} \Delta^2 f^2}{8\pi} \, \mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma] \\
&\quad + \left(\frac{2v^4}{\sqrt{\epsilon}} + \frac{v}{\epsilon^2} \right) \frac{\tilde{\alpha}^2 f^4}{16} \left(\mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma] \right)^2
\end{aligned}
$$

---

[1] The values of b and c are gauge and renormalization prescription dependent. We refer here to their values in the Landau gauge (5) and in the modified minimal subtraction scheme with renormalization scale $\lambda \simeq 2\Delta$.

$$\times \left[\ln\left(\frac{v^2 \tilde{e}^2 f^2 \, \mathrm{Tr}[\Sigma^\dagger, Q][Q, \Sigma]}{8\Delta^2} \right) - \bar{c} - \frac{3}{2} \right].$$

(8)

Whether the electromagnetic corrections computed above can modify the position of the minimum of the potential depends on the hierarchy assumed for the scales μ, Δ, etc.

It is probably instructive to compare the present situation with the simpler one of scalar QED. The effective potential there has the form (omitting numerical factors) [26]

$$V = m^2 \phi^2 + \lambda \phi^4 + (\lambda^2 + \alpha^2)\phi^4 \left(\frac{\alpha \phi^2}{M^2} \right).$$

(9)

In the massless case the minimum of (9) is at $\bar{\phi}^2 = M^2/\alpha e^{-\frac{1}{2} + \frac{\lambda}{\alpha^2}}$. Assuming $\lambda \lesssim \alpha^2$ the term proportional to λ^2 can be disregarded and higher loop corrections are under control at $\phi = \bar{\phi}$. The presence of a finite mass term will not destroy this minimum if, at $\phi = \bar{\phi}$, it is smaller than the other terms. This condition translates into $m^2 e^{-\frac{1}{2} + \frac{\lambda}{\alpha^2}} < \alpha M^2$. In our case, we have mass terms of the order $\alpha \Delta^2$, $(m_d - m_u)m_s \Delta^2/\mu^2$, and $M^2 \sim \Delta^2$. The role of the self-interaction is played by terms coming from the electromagnetic mass term ($\lambda \sim \alpha \Delta^2/\mu^2$), the quark mass term ($\lambda \sim (m_d - m_u)m_s \Delta^2/\mu^4$) and the electromagnetic interaction term ($\lambda \sim \alpha^2$). Assuming $\alpha \sim \Delta^2/\mu^2$, $(m_d - m_u)m_s/\mu^2$ (or larger) the condition for the survival of the non-trivial minimum is satisfied up to numerical factors. Those numerical factors (depending, among other things, on the low energy constants $\bar{b}$ and $\bar{c}$) determine whether the Coleman–Weinberg mechanism occurs. Thus, let us analyze them more carefully in two separate situations.

Asymptotic limit

To analyze the simultaneous condensation of neutral and charged kaons we use the parameterization

$$K_0 = \phi \cos x e^{i\theta_1}, \qquad K^+ = \phi \sin x e^{i\theta_2}.$$

(10)

Due to charge and hypercharge invariance the potential does not depend on the phases θ_1 and θ_2. Isospin breaking effects create a dependence on x. The effec-

tive potential (8) becomes

$$V_{\mathrm{eff}} = (m_d - m_u)\left(\frac{\hat{m} m_s^2 f^2}{4\mu^2} - \frac{a}{8\pi}m_s \Delta^2 \right)\cos 2x$$

$$- \frac{\bar{b}}{8\pi}\left(\frac{2v^2}{\sqrt{\epsilon}} + \frac{1}{v\epsilon^2} \right)\tilde{\alpha}\Delta^2 f^2 \chi$$

$$+ \left(\frac{2v^4}{\sqrt{\epsilon}} + \frac{v}{\epsilon^2} \right)\frac{\tilde{\alpha}^2 f^4}{16}\chi^2 \ln\left(\frac{v^2 e^2 f^2}{8\Delta^2 e^{\bar{c}+\frac{3}{2}}}\chi \right)$$

$$+ \mathcal{O}\left(\frac{(m_d - m_u)^2}{m_s^2} \right),$$

(11)

with

$$\chi = 2\sin^2 x \left(\cos^2 x + \left(1 + \cos\left(\frac{\phi}{\sqrt{2}\,f} \right) \right)\sin^2 x \right)$$

and we approximate $\cos\left(\frac{\sqrt{2}\phi}{f} \right) \simeq 0$.

Let us now consider the limit $\mu \to \infty$. For values of μ such that

$$\delta m m_s \ll \frac{\tilde{\alpha}}{\sqrt{\epsilon}}f^2$$

(12)

(but still satisfying the condition for K^0 condensation in Eq. (2), that is violated only around 10^6 GeV) the first term in Eq. (11) is smaller than the second one and can be disregarded. Numerically, condition (12) is satisfied for $\mu > 3$ GeV. Let us momentarily put aside the second them in Eq. (11) (electromagnetic mass). Minimizing in relation to x we find a solution

$$x \simeq \frac{\Delta e^{\frac{\bar{c}}{2} + \frac{3}{4}}}{vef}.$$

(13)

At this value of x, the electromagnetic mass term can be disregarded compared to the one we kept if

$$-4\bar{b}e^{-(c+\frac{3}{2})} \lesssim 1.$$

(14)

For many, but not all, natural values of $\bar{b}$ and $\bar{c}$ this condition is satisfied and the solution in Eq. (13) can be trusted. Unfortunately, the asymptotic values of these parameters in the limit $\mu \to \infty$ are not known (the computation of $\bar{c}$ involves the calculation of four-loops diagrams) and we cannot determine whether (14). In Fig. 2 we show, as an example, the effective potential for a natural choice of parameter values and very high value of the chemical potential ($\mu = 10$ GeV, $m_u = 4$ MeV, $m_d = 7$ MeV, $m_s = 150$ MeV, $\bar{b} = -1.3$, $\bar{c} = 0.3$). It shows the characteristic shape of a potential with a first order phase transition.
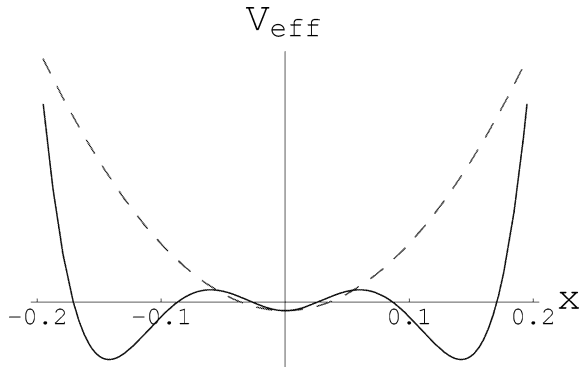
Fig. 2. Effective potential as a function of $x = \tan(|K^+|/|K^0|)$ ($\mu = 10$ GeV, $\bar{b} = -1.3$, $\bar{c} = 0.3$, and leading perturbative results for the remaining parameters). The dashed line shows the effective potential without the electromagnetic contribution.
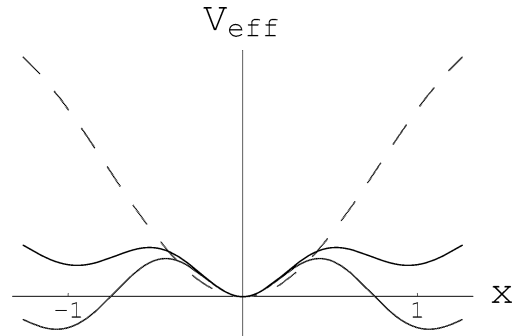

Fig. 3. Effective potential as a function of $x = \tan(|K^+|/|K^0|)$ ($\mu = 500$ MeV, $\Delta = 50$ MeV, $f = 3f_{\text{pert}}$, $\bar{b} = -0.5$, $\bar{c} = 2.5$ (lower curve) and 1.5 (upper curve) and leading perturbative results for the remaining parameters). The dashed line shows the effective potential without the electromagnetic contribution.

"Realistic" densities

For $\mu < 3$ GeV the quark mass terms are no longer negligible compared to the electromagnetic mass terms. In fact, for the densities that may be found in neutron star cores ($\mu \simeq 500$ MeV) it is the dominant mass term for the charged kaons and the one loop effects are too small to overcome it for most values of the parameters. However, at lower densities the values of the low energy constants are not so well determined since the perturbative results do not apply. Some choices for the values of these low energy constants that do not violate the expectations of dimensional analysis result in charged kaon condensation. As an example, we show in Fig. 3 the effective potential for two choices of the parameters.

In both of them the value of the decay constant f was changed from the value suggested from perturbation theory ($f \to 0.6\mu = 3f_{\text{pert}}$) and used $[\bar{b}] = -0.5$. The two solid curves correspond to $\bar{c} = 2.5$ and $\bar{c} = 1.5$. This change in $\bar{c}$ is enough to transform the global minimum into a local minimum. This set of parameters were carefully chosen for most of the parameter space the quark mass term overwhelms the others and there is no charged kaon condensation.

A better idea of the likelihood of charged kaon condensation at these densities can be perhaps obtained through the use of QCD models to estimate the unknown low energy constants in the density range inaccessible to perturbation theory.

We have considered the possibility of charged kaon condensation and (electromagnetic) superconductivity at high density quark matter. At asymptotically high densities, where perturbative QCD applies and the question can be decided on first principles, a complicated computation of some low energy constants are necessary to settle the issue. We find, however, that for most natural values of these constants charged kaon condensation indeed occurs. At lower densities the situation is the opposite. For most reasonable values of the low energy constants the quark mass effects overwhelm the electromagnetic effects and there is no K^+ condensation.

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